

Econ435 – Financial Markets and the Macroeconomy

Problem Set 2

Due: Wednesday, August 1

Question 1

Suppose the risk-free asset gives a rate of return of 4%, and you have information on the returns (as percentages) of the following stocks which are available on the market:

State of the economy	Probability	Stock A	Stock B
Poor	0.3	-2	10
Moderate	0.3	4	-4
Good	0.3	10	4
Excellent	0.1	18	16

- (i) Calculate the mean, variance and standard deviation of the returns on the two stocks.
- (ii) Calculate the covariance and correlation between the returns on the two stocks. (Hint: Work with percentages, rather than decimals. You should get $E(r_A) = 5.4\%$, $E(r_B) = 4.6\%$, $\sigma_A^2 = 39.24$, $\sigma_B^2 = 44.04$, $\rho_{AB} = 0.12$. If you don't, just use these values for the rest of the questions.)
- (iii) Draw (approximately) the portfolio opportunity set and the Capital Allocation Line.
- (iv) Investor X has a coefficient of risk aversion $A = 6$ and an optimal risky portfolio with proportions $w_A = 0.78$ and $w_B = 0.22$ in stocks A and B, respectively. Calculate the optimal proportion (y^*) to be invested in the risky portfolio and the mean, variance and standard deviation of the complete portfolio (the portfolio that includes both the risky portfolio and the risk-free asset). Show graphically the optimal complete portfolio, using indifference curves, the portfolio opportunity set and the CAL. (Hint: First calculate the mean and variance of the optimal risky portfolio. You should get $E(r_p) = 5.22\%$ and $\sigma_p^2 = 27.78\%$. Next, use these numbers to calculate y^* and then use this in the formulas for the complete portfolio.)
- (v) How would the CAL change if the borrowing rate was different from (i.e., higher than) the lending rate? How would the optimal complete portfolio of investor X change?
- (vi) (*extra credit*) Suppose that, in addition to the situation described in part (iv), the market offers another risk-free asset with a rate of return equal to 3%. How is investor X going to divide the investment between the three assets (the original risk-free asset, the new risk-free asset and the risky portfolio)? How do the mean, variance and standard deviation of the optimal complete portfolio change?