Answer Key

1. C
2. E
3. D
4. E
5. A
6. E
7. D
8. C
9. B
10. A
11. D
12. B
13. C
14. D
15. B
16. B
17. B
18. D
19. D
20. A
21. B
22. A
23. E
24. B
25. A
26. A
27. B
28. B
29. B
30. D

Question 1 (*20 points*)

(i) Define both open- and closed-end managed investment companies, making sure to enumerate the differences between them.

Answer: Both open-end and closed-end managed investment companies are financial intermediaries that collect funds from individual investors and use them to create and manage a portfolio, for an annual fee. The differences between them stem from two sources:

(1) number of outstanding shares:

   (i) *open-end managed investment companies* stand ready to redeem or issue new shares at any time, meaning that the number of outstanding shares changes continually.

   (ii) *closed-end managed investment companies* do not redeem or issue shares after the initial issue, therefore the number of shares is fixed.

(2) price of shares:

   (i) *open-end managed investment companies* trade directly with the investors, so they issue and redeem shares at the NAV (barring any load the fund would carry).

   (ii) *closed-end managed investment companies* do not deal with the investors after the initial issue, so investors need to trade their holdings on the market; hence, the price of the shares can (and usually does) deviate from the NAV.

Question 2 (*20 points*)

The evolution of the stock market is shown in the table below:

	Year 0		Year 1	
	Price	No. of shares	Price	No. of shares
Stock A	15	500	14	500
Stock B	7	800	8	800
Stock C	32	100	15	200

(i) Is there a stock for which a stock-split occurs? If yes, which one and into how many shares does it split?

Answer: Since the number of shares of stock C changes from 100 in year 0 to 200 in year 1, it is apparent that a 2-for-1 split occurred for stock C.

(ii) Calculate the price-weighted index for years 0 and 1.

Answer: The value of the *price-weighted index in year* 0 is

$$\boxed{I_0^p = \frac{15+7+32}{3} = \frac{54}{3} = 18.}$$

To calculate the value of the price-weighted index in year 1, we first need to calculate the *divisor* (because of the 2-for-1 stock split for stock C):

$$\frac{15+7+\frac{32}{2}}{d} = 18 \quad \Rightarrow \quad \boxed{d = \frac{15+7+\frac{32}{2}}{18} = \frac{15+7+16}{18} = \frac{38}{18} = 2.111.}$$

Now we can calculate the *price-weighted index in year 1*:

$$\boxed{I_1^p = \frac{14+8+15}{d} = \frac{37}{2.111} = 17.526.}$$

The *value-weighted index in year 1*, based on a value of $1,000$ points for year 0, is

$$\boxed{I_1^v = \frac{14 \cdot 500 + 8 \cdot 800 + 15 \cdot 200}{15 \cdot 500 + 7 \cdot 800 + 32 \cdot 100} \cdot 1,000 = \frac{16,400}{16,300} \cdot 1,000 = 1,006.13 \text{ points.}}$$

Question 3 (*20 points*)

An investor's optimal risky portfolio P offers a rate of return of 8% and has a standard deviation $\sigma_p = 12\%$. Her coefficient of risk aversion is $A = 4$. The market offers a risk-free asset with a rate of return of 4%.

(i) What proportion of the investor's wealth is invested in the risky portfolio (y^*)?

Answer: The investor would calculate the fraction to be put in the risky portfolio as follows:

$$\boxed{y^* = \frac{E(r_p) - r_f}{0.01 \cdot A \cdot \sigma_p^2} = \frac{8-4}{0.01 \cdot 4 \cdot 12^2} = \frac{4}{5.76} = 0.69 = 69\%.}$$

(ii) What are the complete portfolio's expected return, variance and standard deviation?

Answer: Using y^*, we can calculate the expected return and the risk of the complete portfolio:

$$\boxed{E(r_c) = y^* E(r_p) + (1-y^*) r_f = 0.69 \cdot 8 + (1-0.69) \cdot 4 = 6.78\%,}$$

$$\boxed{\sigma_c = y^* \cdot \sigma_p = 0.69 \cdot 12 = 8.33\%,}$$

$$\boxed{\sigma_c^2 = (\sigma_c)^2 = 8.33^2 = 69.44.}$$

(iii) Draw the portfolio opportunity set, the CAL and show where the complete portfolio lies on the CAL.

Answer: More precisely, you would need to draw (roughly) the minimum-variance frontier and the CAl, which is tangent to it, and show that the optimal complete portfolio lies between r_f and P.

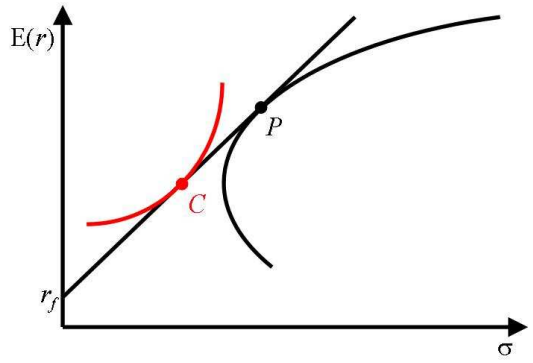

Figure 1: Minimum Variance Frontier, POS, CAL and the Optimal Complete Portfolio.

Question 4 (*extra 10 points*)

Draw the minimum variance frontier and explain what it plots. Show how the shape of the CAL changes when the borrowing rate is higher than the lending rate, and detail on how a borrowing investor would be affected (as compared to the case when the lending rate and the borrowing rate are equal).

Answer: The minimum variance frontier plots the portfolios with the lowest risk for a given expected return, that can be constructed with the existing assets (hence the name "minimum variance frontier"). Its shape is shown in the graph below.

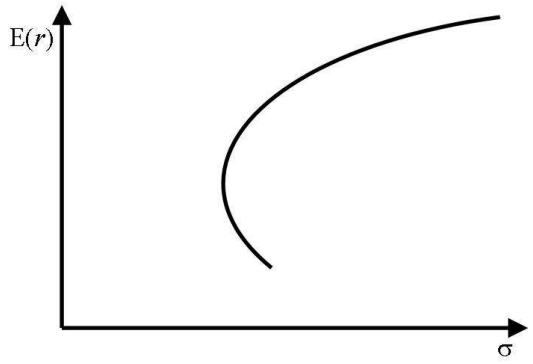

Figure 2: Minimum Variance Frontier.

When the borrowing rate is higher than the lending rate, the CAL is not a straight line anymore, but rather as shown in the graph below: a straight line, followed by a portion of the minimum-variance frontier, and then another straight line..

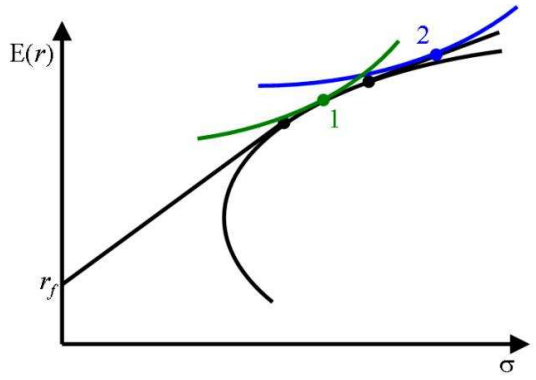

Figure 3: The CAL when the borrowing rate is higher than the lending rate.

A initially borrowing investor can now be in two situation. Investor 1, while desiring to borrow, finds the borrowing rate too high and decides to invest *only in risky assets*. Thus, the optimal complete portfolio of investor 1 lies on the minimum variance frontier, as shown in figure 3. Investor 2, on the other hand, still decides to borrow and, as such, will have an optimal complete portfolio that lies on the second line segment that comprises the CAL.

Both optimal complete portfolios will lie on lower indifference curves than if the borrowing rate were the same as the lending rate. This was to be expected, as the investors would need to spend more to achieve the same investment as before.