Option Valuation

Chapter 21

Intrinsic and Time Value

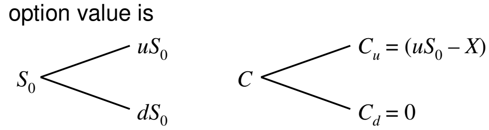
- *intrinsic value* of in-the-money options = the payoff that could be obtained from the immediate exercise of the option
  - for a call option: stock price – exercise price
  - for a put option: exercise price – stock price
- the intrinsic value for out-the-money or at-the-money options is equal to 0
- *time value* of an option = difference between actual call price and intrinsic value
- as time approaches expiration date, time value goes to zero

Determinants of Option Values

	Call	*Put*
Stock price	+	–
Exercise price	–	+
Volatility of stock price	+	+
Time to expiration	+	+
Interest rate	+	–
Dividend rate of stock	–	+

Binomial Option Pricing

- consider a stock that currently sells at S_0
- the price an either increase by a factor u or fall by a factor d (probabilities are irrelevant)
- consider a call with exercise price X such that $dS_0 < X < uS_0$
- hence, the evolution of the price and of the call option value is



Binomial Option Pricing (cont.)

- now, consider the payoff from writing one call option and buying H shares of the stock, where

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{uS_0 - X}{uS_0 - dS_0}$$

- the value of this investment at expiration is

	Up	Down
Payoff of stock	HuS_0	HdS_0
Payoff of calls	$-(uS_0 - X)$	0
Total payoff	HdS_0	HdS_0

Binomial Option Pricing (cont.)

- hence, we obtained a risk-free investment with end value HdS_0
- arbitrage argument: the current value of this investment should be equal to its present discounted value using the risk-free rate
- H is called the *hedge ratio* (the ratio of the range of call option payoffs and the range of the stock price)
- the argument is based on perfect hedging, or *replication* (the payoff of the investment replicates a risk-free bond)

Binomial Option Pricing – Algorithm

1. given the end of period stock prices, uS_0 and dS_0, calculate the payoffs of the call option, C_u and C_d
2. find the hedge ratio $H = (C_u - C_d)/(uS_0 - dS_0)$
3. calculate HdS_0, the end-of-year certain value of the portfolio including H shares of the stock and one written call
4. find the present value of HdS_0, given the riskfree interest rate r
5. calculate the price of the call using the arbitrage argument:

   $HS_0 - C = PV(HdS_0)$

Binomial Option Pricing – Example

- $S_0 = 100$
- $d = .75$
- $u = 1.5$
- $X = 120$
- $r = 5\%$

1. $uS_0 = 150$, $dS_0 = 75$

   $C_u = uS_0 - X = 30$, $C_d = 0$
2. $H = (C_u - C_d) / (uS_0 - dS_0) = 0.4$
3. $HdS_0 = 30$
4. $PV(HdS_0) = HdS_0 / (1 + r) = 28.57$
5. $HS_0 = 0.4 \cdot 100 = 40$

   $C = HS_0 - PV(HdS_0) = 11.43$

Generalized Binomial Option Pricing

- the binomial model can be expanded to more than one period
- in this case, we would need to find the hedging ratio H at every node in the tree
- thus, we can construct, at each point in time, a perfectly hedged portfolio – *dynamic hedging*
- some of the nodes will be shared by different branches (e.g., the "up and down" scenario would yield the same price as the "down and up" scenario)
- although numerous and tedious calculations, can "easily" program into a computer

Black-Scholes Valuation Model

- Assumptions
  - European call option
  - underlying asset does not pay dividends until expiration date
  - both the (riskfree) interest rate r and the variance of the return on the stock σ^2 are constant
  - stock prices are continuous (no sudden jumps)

Black-Scholes Valuation Model (cont.)

- Formula
  - the current price of the call option is
    $C_0 = S_0\, N(d_1) - X\, e^{-rT}\, N(d_2)$
    where:
    - S_0 is the current price of the stock
    - X is the exercise price
    - T is the time until maturity of option (in years)
    - $e = 2.71828$ is the base of the natural logarithm
    - $N(\cdot)$ is the probability from a standard normal distribution
    - $d_1 = \dfrac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

Black-Scholes Formula – Example

- $S_0 = 100$
- $X = 95$
- $r = 10\%$ per year
- $T = 0.25$ years (one quarter)
- $\sigma = 0.50$ (50% per year)
- $d_1 = \dfrac{\ln(100/95) + (0.10 + 0.5^2/2)0.25}{0.5\sqrt{0.25}} = 0.43$
- $d_2 = 0.43 - 0.5\sqrt{0.25} = 0.18$
- $N(d_1) = N(0.43) = 0.6664, \quad N(d_2) = N(0.18) = 0.5714$
- $C_0 = 100 \cdot 0.6664 - 95 \cdot e^{-0.10 \cdot 0.25} \cdot 0.5714 = \$13.70$

Black-Scholes Formula – Put Options

- to find the value of a European put option, we can use the put-call parity theorem:

  $P_0 = C_0 - S_0 + PV(X)$

  where the present value of X is calculated in continuous time:

  $PV(X) = X\, e^{-rT}$
- this yields the formula:

  $P_0 = X\, e^{-rT}\, [1 - N(d_2)] - S_0\, [1 - N(d_1)]$

Implied Volatility

- the Black-Scholes formula is based on four observed variables (S_0, X , T and r) and one unobserved variable (σ)
- we can estimate σ from historical data
- alternatively, we can calculate the value of σ that equates the Black-Scholes value of a call to the observed value of a call ➔ *implied volatility*
- investors would buy the call option if they think the actual standard deviation of the stock is higher than the implied volatility

Delta

- the *delta* of an option is the change in the price of an option due to a $1 increase in the stock price
- it summarizes the exposure to stock price risk
- it is the same as the hedge ratio in the binomial model
- for a call option, $delta = N(d_1) > 0$
- for a put option, $delta = N(d_1) - 1 < 0$