

Chapter 4

Understanding Interest Rates

Present Value

- Concept
 - a dollar today is (in general) more valuable than a dollar tomorrow – you can invest this dollar (e.g., deposit it in a bank) so that tomorrow you have more than \$1
- Example: Simple loan of \$1 at 10% interest

Year	1	2	3	...	n
	\$1.10	\$1.21	\$1.33		$\$1 \times (1 + i)^n$

$$\text{PV of future } \$x = \frac{x}{(1+i)^n}$$

This is called *discounting the future*

Present Value – Example 2

- suppose you won the lottery and are promised \$20 million in payments of \$1 million over the next 20 years
- what you get is actually less than \$20million, because the present value of these payments is

$$PV = \frac{\$1mil}{(1+i)} + \frac{\$1mil}{(1+i)^2} + \dots + \frac{\$1mil}{(1+i)^{20}}$$

- if we assume that $i = 10\%$, the present value of your lottery prize is actually \$9.4 million!

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Credit Instruments

- Loans
 - *simple loan* = the borrower must repay the funds loaned at maturity date, along with an additional payment for the interest (e.g., commercial loans to businesses)
 - *fixed-payment (fully amortized) loan* = the borrower makes the same payment every period (principal plus interest) for a set number of years (e.g., auto loans, mortgages)

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Credit Instruments (cont.)

■ Bonds

- *coupon bond* = the owner (i.e., lender) receives a fixed interest payment (called *coupon payment*) every period until maturity, when the final amount (called *face value* or *par value*) is repaid (e.g., Treasury bonds, corporate bonds)
- *discount (zero coupon) bond* = bought at a price below its face value, while the face value is repaid at maturity (no interest payments)

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Yield to Maturity

- *yield to maturity (i)* = interest rate that equates today's value with present value of all future payments

■ Loans:

- simple loan, \$100 for 1 year, 10% interest rate:

$$\$100 = \frac{\$110}{(1+i)} \Rightarrow i = \frac{\$110 - \$100}{\$100} = 10\% = \text{interest rate}$$

- fixed-payment loan – more complicated:

$$\text{Loan value} = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \dots + \frac{FP}{(1+i)^n}$$

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Yield to Maturity

- Coupon bond:

- same strategy as fixed-payment loan, using price P as value of loan

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \dots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

where F is the face value of the bond and

$C = r \times F$ is the yearly coupon payment (r is the coupon rate)

- for a *consol (perpetuity)*, i.e. a coupon bond with no maturity date and no payment of principal,

$$P = \frac{C}{i} \Rightarrow i = \frac{C}{P}$$

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Yield to Maturity (cont.)

- Discount bond:

- same strategy as simple loan, using price as value of loan

$$P = \frac{F}{(1+i)^n}$$

- for a one-year discount bond,

$$P = \frac{F}{1+i} \Rightarrow i = \frac{F-P}{P}$$

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Yield to Maturity (cont.)

- note that in the case of bonds, the yield to maturity is not necessarily equal to coupon rate
- also, for both consols and discount bonds, the yield to maturity is negatively related to price
- it can be shown that this relationship holds also for general coupon bonds
- hence, current bond prices and interest rates are negatively related
- when the bond is at par (i.e., price = face value), the yield to maturity and coupon rate are equal

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Relationship Between Price and Yield to Maturity

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three Interesting Facts in Table 1

1. When bond is at par, yield equals coupon rate
2. Price and yield are negatively related
3. Yield greater than coupon rate when bond price is below par value

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Current Yield

- yield to maturity is the most accurate measure of interest rates (and it is what economists mean by “interest rates”)
- yield to maturity is difficult to calculate – other useful approximations are easier
- *current yield* is calculated for coupon bonds:
$$i_c = \frac{C}{P} \Rightarrow \text{negatively related to price}$$
- it is a better approximation to yield to maturity, the nearer price is to par and the longer is the maturity of the bond
- the change in current yield always signals change in same direction as yield to maturity

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Yield on a Discount Basis

- is calculated for discount bonds:
$$i_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}}$$
- again this is negatively related to price
- it understates yield to maturity – the longer the maturity, the greater is the understatement
- the change in discount yield always signals a change in the same direction for the yield to maturity

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Distinction Between Interest Rates and Returns

- the interest rate (yield to maturity) is *not* the same as the rate of return from buying the bond
- *rate of return* = total net benefit from investment (payments to the owner plus change in value) divided by the initial cost
- the rate of return for a bond held from t to $t + 1$ is

$$R = \frac{P_{t+1} - P_t + C}{P_t} = \frac{P_{t+1} - P_t}{P_t} + \frac{C}{P_t} = g + i_c$$

where g is the rate of capital gain

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Table 2 Analysis

- table 2 looks at the one-year return on several 10%-coupon-rate bonds purchased at par
- the interest rate (yield to maturity) changes from 10% to 20% during the year
- the “price next year” is calculated using the formula of the yield to maturity for coupon bonds
- **Conclusions:**
 - prices and returns are more volatile for long-term bonds because they have higher interest-rate risk
 - there is no interest-rate risk for any bond whose maturity equals holding period

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Key Facts about the Relationship between Interest Rates and Returns

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

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Key Findings from Table 2

- the only bond whose return equals the yield is the one with maturity equal to the holding period
- for bonds with maturity longer than the holding period, $i \uparrow$, $P \downarrow$, implying a capital loss
- the longer is maturity, the greater is the percentage price change associated with the interest rate change
- the longer is maturity, the more return changes with change in the interest rate
- a bond with high initial interest rate can still have a negative return if $i \uparrow$

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Real Interest Rate

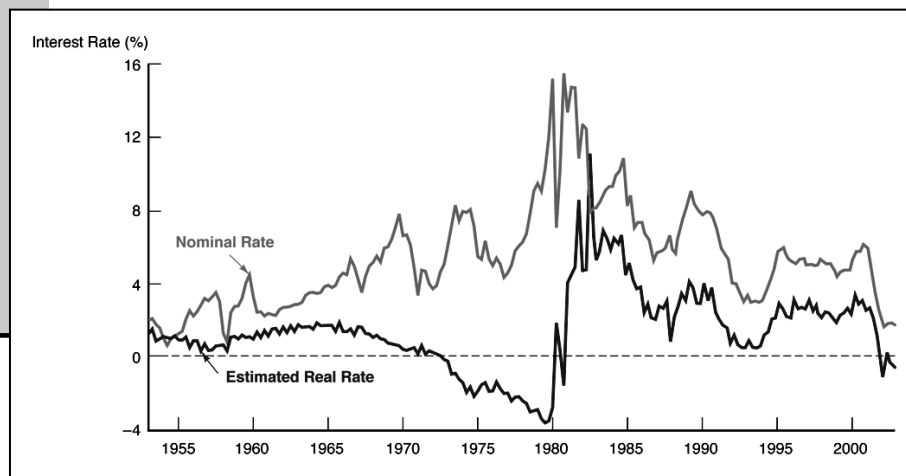
- *real interest rate* = interest rate that is adjusted for expected changes in the price level (i.e., expected inflation, π^e)

$$i_r = i - \pi^e$$

- the real interest rate reflects more accurately the true cost of borrowing
- when the real rate is low, there are greater incentives to borrow and less to lend
- it can be (and sometimes is) negative, if expected inflation is higher than the nominal interest rate

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U.S. Real and Nominal Interest Rates



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